

$D_s D^*$ MOLECULE AS AN AXIAL MESON

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We use QCD sum rules to study the possible existence of a $D_s \bar{D}^* + D_s^* \bar{D}$ molecule with the quantum number $J^P = 1^+$. We consider the contributions of condensates up to dimension eight and work at leading order in α_s . We obtain $m_{D_s D^*} = (3.96 \pm 0.10)$ GeV around 100 MeV above the mass of the meson $X(3872)$. The proposed state is a natural generalized state to the strangeness sector of the $X(3872)$, which was also found to be consistent with a multiquark state from a previous QCD sum rule analysis.

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The recent appearance of new resonances observed by BaBar, BELLE, CLEO and FOCUS collaborations, and partly confirmed by the Fermilab experiments CDF and D0, sheds new light on the spectroscopy of charmonium states. Among these new mesons, some have their masses very close to the meson-meson threshold like the $X(3872)$ [1] and the $Z^+(4430)$ [2]. Of special importance is the appearance of the $Z^+(4430)$, which decays into ψ' and π^+ and, therefore, can not be described as ordinary $c\bar{c}$ meson. Its nature is completely open, but an intriguing possibility is the interpretation as tetraquark state or molecular state [3, 4, 5]. If these mesons are really molecular states, then many other molecules should exist. A systematic study of these molecular states and their experimental observation would confirm its structure and provide a new testing ground for QCD within multiquark configuration. In this context a natural extension would be to probe the strangeness sector. In particular, in analogy with the meson $X(3872)$, a $D_s D^*$ molecule with $J^P = 1^+$ would decay into $J/\psi K^* \rightarrow J/\psi K \pi$ and, therefore, could be easily reconstructed. Here we use the QCD sum rules [6, 7, 8] to predict the mass of such a molecular state.

This calculation is of particular importance for new upcoming experiments which can investigate with much higher precision the charmonium energy regime, like the PANDA experiment at the antiproton-proton facility at FAIR, or a possible Super-B factory experiment. Especially PANDA can do a careful scan of the various thresholds being present, in addition to precisely going through the exact form of the resonance curve.

In a previous calculation, the QCDSR approach was used to study the $X(3872)$ considered as a diquark-antidiquark state [9], and the $Z^+(4430)$ meson, considered as a $D^* D_1$ molecular state [5]. In both cases a very good agreement with the experimental mass was obtained.

Considering a $D^* D_s$ molecule with $J^P = 1^+$, a possible current describing such state is given by:

$$j_\mu = \frac{i}{\sqrt{2}} [(\bar{s}_a \gamma_5 c_a)(\bar{c}_b \gamma_\mu d_b) - (\bar{s}_a \gamma_\mu c_a)(\bar{c}_b \gamma_5 d_b)] , \quad (1)$$

where a and b are color indices. We have considered the anti-symmetrical state $D^{*-} D_s^{*+} - D^- D_s^{*+}$ to keep a closer relation with the $X(3872)$ meson. Considering the $X(3872)$ as a $D^* D$ molecule, the combination $D^{*0} \bar{D}^0 - D^0 \bar{D}^{*0}$ has $J^{PC} = 1^{++}$ as the $X(3872)$ meson. Of course the symmetrical combination: $D^{*-} D_s^{*+} + D^- D_s^{*+}$ would provide exactly the same mass, within our sum rule approach.

The sum rule is constructed from the two-point correlation function:

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j_\mu(x) j_\nu^\dagger(0)] | 0 \rangle = -\Pi_1(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \Pi_0(q^2) \frac{q_\mu q_\nu}{q^2}. \quad (2)$$

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Since the axial vector current is not conserved, the two functions, Π_1 and Π_0 , appearing in Eq. (2) are independent. They have respectively the quantum numbers of the spin 1 and 0 mesons. Therefore, we choose to work with the Lorentz structure $g_{\mu\nu}$, since it gets contributions only from the 1^+ state.

On the OPE side, we work at leading order in α_s in the operators and consider the contributions from condensates up to dimension eight. The correlation function, Π_1 , in the OPE side can be written as a dispersion relation:

$$\Pi_1^{OPE}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2} + \Pi_1^{m_s \langle \bar{q}q \rangle^2}(q^2) + \Pi_1^{mix \langle \bar{q}q \rangle}(q^2), \quad (3)$$

where $\rho^{OPE}(s)$ is given by the imaginary part of the correlation function: $\pi \rho^{OPE}(s) = \text{Im}[\Pi_1^{OPE}(s)]$. We get:

$$\rho^{OPE}(s) = \rho^{pert}(s) + \rho^{m_s}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{m_s \langle \bar{q}q \rangle}(s) + \rho^{\langle G^2 \rangle}(s) + \rho^{mix}(s) + \rho^{\langle \bar{q}q \rangle^2}(s) + \rho^{m_s \cdot mix}(s) + \rho^{m_s \langle \bar{q}q \rangle^2}(s), \quad (4)$$

with

$$\begin{aligned} \rho^{pert}(s) &= \frac{3}{2^{12}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)(1+\alpha+\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s]^4, \\ \rho^{m_s}(s) &= -\frac{3m_s m_c}{2^{10}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \frac{d\beta}{\beta^2} (1-\alpha-\beta)(3+\alpha+\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s]^3, \\ \rho^{\langle \bar{q}q \rangle}(s) &= -\frac{3m_c}{2^8\pi^4} (\langle \bar{q}q \rangle + \mathfrak{B}) \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta} (1+\alpha+\beta) [(\alpha+\beta)m_c^2 - \alpha\beta s]^2, \\ \rho^{m_s \langle \bar{q}q \rangle}(s) &= \frac{3m_s}{2^8\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \left[\frac{((\alpha+\beta)m_c^2 - \alpha\beta s)^2}{1-\alpha} \mathfrak{B} + \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta} ((\alpha+\beta)m_c^2 - \alpha\beta s) (4m_c^2 \langle \bar{q}q \rangle \right. \\ &\quad \left. - ((\alpha+\beta)m_c^2 - \alpha\beta s) \mathfrak{B}) \right], \\ \rho^{\langle G^2 \rangle}(s) &= \frac{\langle g^2 G^2 \rangle}{2^{11}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} [(\alpha+\beta)m_c^2 - \alpha\beta s] \left[\frac{m_c^2(1-(\alpha+\beta)^2)}{\beta} - \frac{(1-2\alpha-2\beta)}{\alpha} [(\alpha+\beta)m_c^2 - \alpha\beta s] \right], \\ \rho^{mix}(s) &= \frac{3m_c m_0^2}{2^9\pi^4} (\langle \bar{q}q \rangle + \mathfrak{B}) \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \left[-\frac{2}{\alpha} (m_c^2 - \alpha(1-\alpha)s) + \int_{\beta_{min}}^{1-\alpha} d\beta [(\alpha+\beta)m_c^2 - \alpha\beta s] \left(\frac{1}{\alpha} + \frac{2(\alpha+\beta)}{\beta^2} \right) \right], \\ \rho^{\langle \bar{q}q \rangle^2}(s) &= \frac{m_c^2 \langle \bar{q}q \rangle \mathfrak{B}}{2^4\pi^2} \sqrt{1-4m_c^2/s}, \\ \rho^{m_s \cdot mix}(s) &= \frac{m_s m_0^2}{2^8\pi^4} \left\{ \sqrt{1-4m_c^2/s} \left(6m_c^2 \langle \bar{q}q \rangle + \frac{m_c^2 + 2s}{3} \mathfrak{B} \right) - \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \left[3m_c^2 \langle \bar{q}q \rangle \left(\frac{1}{\alpha} - \frac{m_c^2}{\alpha s - m_c^2} \right) + \right. \right. \\ &\quad \left. \left. \frac{m_c^2}{1-\alpha} \mathfrak{B} - 2\mathfrak{B} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta} [(\alpha+\beta)m_c^2 - \alpha\beta s] \right] \right\}, \\ \rho^{m_s \langle \bar{q}q \rangle^2}(s) &= \frac{3m_s m_c \langle \bar{q}q \rangle \mathfrak{B}}{2^7\pi^2} \sqrt{1-4m_c^2/s}, \\ \Pi_1^{m_s \langle \bar{q}q \rangle^2}(q^2) &= -\frac{m_s m_c^3 \langle \bar{q}q \rangle \mathfrak{B}}{2^5\pi^2} \int_0^1 d\alpha \frac{1-\alpha}{(m_c^2 - \alpha(1-\alpha)q^2)}, \\ \Pi_1^{mix \langle \bar{q}q \rangle}(q^2) &= -\frac{m_c^2 m_0^2 \langle \bar{q}q \rangle \mathfrak{B}}{2^5\pi^2} \int_0^1 d\alpha \frac{\alpha(1-\alpha)}{m_c^2 - \alpha(1-\alpha)q^2} \left[1 + \frac{m_c^2}{m_c^2 - \alpha(1-\alpha)q^2} - \frac{1}{1-\alpha} \right]. \end{aligned} \quad (5)$$

where the integration limits are given by $\alpha_{min} = (1 - \sqrt{1 - 4m_c^2/s})/2$, $\alpha_{max} = (1 + \sqrt{1 - 4m_c^2/s})/2$, $\beta_{min} = \alpha m_c^2/(s\alpha - m_c^2)$, and we have used $\langle \bar{q}g\sigma.Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $\langle \bar{s}G\sigma.GS \rangle = m_0^2 \beta$.

It is very interesting to notice that the current in Eq. (1) has a OPE behavior very similar to the scalar-diquark axial-antidiquark current used for the $X(3872)$ meson in ref. [9].

In the phenomenological side, we write a dispersion relation to the correlation function in Eq. (2):

$$\Pi_1^{phen}(q^2) = \int ds \frac{\rho^{phen}(s)}{s - q^2} + \dots, \quad (6)$$

where $\rho^{phen}(s)$ is the spectral density and the dots represent subtraction terms. The spectral density is described, as usual, as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states:

$$\rho^{phen}(s) = \lambda^2 \delta(s - m_{D_s D^*}^2) + \rho^{cont}(s), \quad (7)$$

where λ gives the coupling of the current to the meson $D_s D^*$:

$$\langle 0 | j | D_s D^* \rangle = \lambda. \quad (8)$$

For simplicity, it is assumed that the continuum contribution to the spectral density, $\rho^{cont}(s)$ in Eq. (7), vanishes below a certain continuum threshold s_0 . Above this threshold, it is assumed to be given by the result obtained with the OPE. Therefore, one uses the ansatz [10]

$$\rho^{cont}(s) = \rho^{OPE}(s) \Theta(s - s_0), \quad (9)$$

After making a Borel transform to both sides of the sum rule, and transferring the continuum contribution to the OPE side, the sum rules for the pseudoscalar meson Z^+ , up to dimension-eight condensates, can be written as:

$$\lambda^2 e^{-m_{D_s D^*}^2/M^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M^2} \rho^{OPE}(s) + \Pi^{m_s \langle \bar{q}q \rangle^2}(M^2) + \Pi^{mix \langle \bar{q}q \rangle}(M^2), \quad (10)$$

To extract the mass $m_{D_s D^*}$ we take the derivative of Eq. (10) with respect to $1/M^2$, and divide the result by Eq. (10).

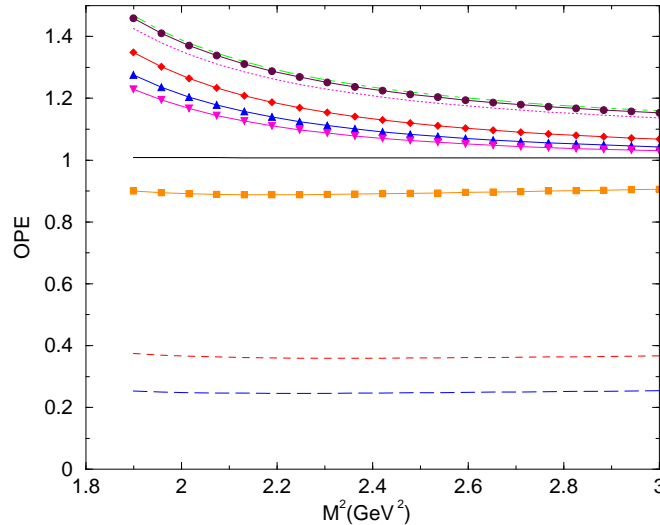


FIG. 1: The OPE convergence in the region $1.9 \leq M^2 \leq 3.0$ GeV² for $\sqrt{s_0} = 4.5$ GeV. We start with the relative perturbative contribution (long-dashed line), and each other line represents the relative contribution after adding of one extra condensate in the expansion: $+m_s$ (dashed line), $+\langle \bar{q}q \rangle$ (dotted line), $+m_s \langle \bar{q}q \rangle$ (dot-dashed line), $+\langle g^2 G^2 \rangle$ (line with circles), $+m_0^2 \langle \bar{q}q \rangle$ (line with squares), $+\langle \bar{q}q \rangle^2$ (line with diamonds), $+m_s m_0^2 \langle \bar{q}q \rangle$ (line with triangles up), $+m_s \langle \bar{q}q \rangle^2$ (line with triangles down), $+m_0^2 \langle \bar{q}q \rangle^2$ (solid line).

The values used for the quark masses and condensates are [8, 11]: $m_c(m_c) = (1.23 \pm 0.05)$ GeV, $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3$ GeV³, $\beta = 0.8 \langle \bar{q}q \rangle$, $m_0^2 = 0.8$ GeV², $\langle g^2 G^2 \rangle = 0.88$ GeV⁴.

We evaluate the sum rules in the Borel range $1.9 \leq M^2 \leq 3.5 \text{ GeV}^2$, and in the s_0 range $4.4 \leq \sqrt{s_0} \leq 4.6 \text{ GeV}$.

From Fig. 1 we see that we obtain a reasonable OPE convergence for $M^2 \geq 1.9 \text{ GeV}^2$. Therefore, we fix the lower value of M^2 in the sum rule window as $M_{min}^2 = 1.9 \text{ GeV}^2$. We notice that the OPE convergence in this case is similar to the OPE convergence for the $X(3872)$ meson [9], and not so good as the OPE convergence for the $Z^+(4430)$ meson [5].

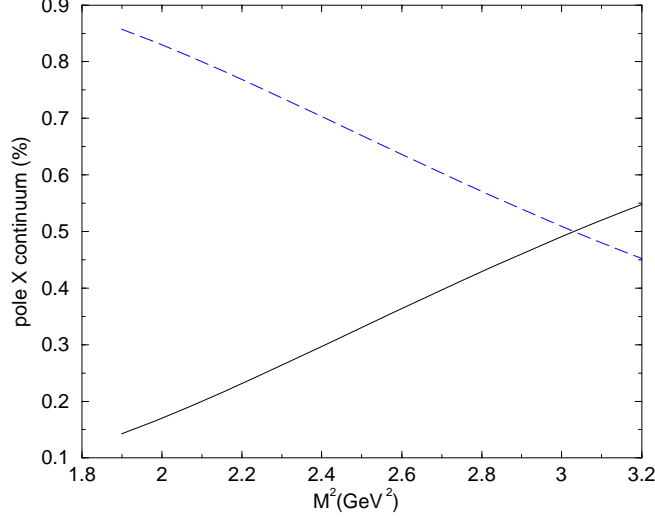


FIG. 2: The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution for $\sqrt{s_0} = 4.5 \text{ GeV}$.

To get an upper limit constraint for M^2 we impose that the QCD continuum contribution should be smaller than the pole contribution. The comparison between pole and continuum contributions for $\sqrt{s_0} = 4.5 \text{ GeV}$ is shown in Fig. 2. From this figure we see that the pole contribution is bigger than the continuum for $M^2 \leq 3.0 \text{ GeV}^2$. The maximum value of M^2 for which this constraint is satisfied depends on the value of s_0 . The same analysis for the other values of the continuum threshold gives $M^2 \leq 2.8 \text{ GeV}^2$ for $\sqrt{s_0} = 4.4 \text{ GeV}$ and $M^2 \leq 3.2 \text{ GeV}^2$ for $\sqrt{s_0} = 4.6 \text{ GeV}$. In our numerical analysis, we shall then consider the range of M^2 values from 1.9 GeV^2 until the one allowed by the pole dominance criteria given above.

In Fig. 3, we show the $D_s D^*$ meson mass, for different values of $\sqrt{s_0}$, in the relevant sum rule window, with the upper validity limits indicated. From this figure we see that the results are very stable as a function of M^2 .

Using the Borel window, for each value of s_0 , to evaluate the mass of the $D_s D^*$ meson and then varying the value of the continuum threshold in the range $4.4 \leq \sqrt{s_0} \leq 4.6 \text{ GeV}$, we get

$$m_{D_s D^*} = (3.97 \pm 0.08) \text{ GeV}, \quad (11)$$

around 100 MeV bigger than the mass of the $X(3872)$ meson. The Borel curve for the mass is quite stable and has a minimum within the relevant Borel window. Such stable Borel curve strongly suggests that there is indeed a very well defined ground state.

To check the dependence of our results with the value of the charm quark mass, we fix $\sqrt{s_0} = 4.5 \text{ GeV}$ and vary the charm quark mass in the range $m_c = (1.23 \pm 0.05) \text{ GeV}$. Using $1.9 \leq M^2 \leq 3.0 \text{ GeV}^2$ we get: $m_{D_s D^*} = (3.96 \pm 0.10) \text{ GeV}$, in agreement with the result in Eq. (11). Therefore, we conclude that the most important sources of uncertainty in our calculation are the values of the continuum threshold and the charm quark mass.

In conclusion, we have presented a QCDSR analysis of the two-point function for a possible $D_s \bar{D}^* + D_s^* \bar{D}$ molecular state with $J^P = 1^+$. This state would decay into $J/\psi K^* \rightarrow J/\psi K \pi$ and, therefore, could be easily reconstructed. Our finding strongly suggests the possibility of the existence of such molecular resonance, whose structure is similar to the $X(3872)$, with a mass about 100 MeV above that of $X(3872)$.

The mass of our proposed state is also close to the newly discovered $X(3940)$ [12] and $Z(3930)$ [13]. While our proposed state has strangeness, the two states do not. Moreover, while $X(3940)$ was found in

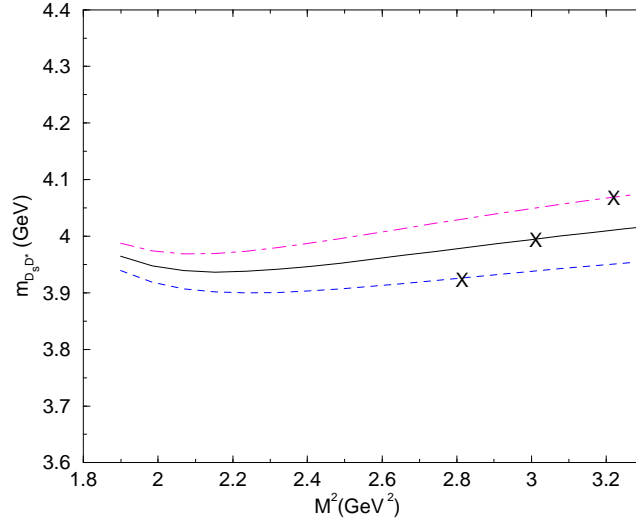


FIG. 3: The $D_s D^*$ meson mass as a function of the sum rule parameter (M^2) for $\sqrt{s_0} = 4.4$ GeV (dashed line), $\sqrt{s_0} = 4.5$ GeV (solid line) and $\sqrt{s_0} = 4.6$ GeV (dot-dashed line). The crosses indicate the upper limit in the Borel region allowed by the dominance of the QCD pole contribution.

the decay of $D^* \bar{D}$ and is a candidate for $\eta_c''(3^1S_0)$, $Z(3930)$ was found in $D \bar{D}$ and is consistent with χ'_{c2} . Therefore, a comparative study with these particles can give us clues on the structures of $X(3872)$ and our proposed state.

As a final remark, one can notice that if we take $m_s = 0$, one obtains the sum rule for the $X(3872)$ meson, considered as a molecular state $D^0 \bar{D}^{*0} - D^{*0} \bar{D}^0$. The results are very similar to the ones obtained in ref. [9], but more stable as a function of the Borel mass. Using $\sqrt{s_0} = (4.4 \pm 0.1)$ GeV one obtains $m_X = (3.88 \pm 0.06)$ GeV, in an excellent agreement with the experimental value.

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